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On consistent truncations in $\mathcal{N} = 2^*$ holography

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ABSTRACT: Although Pilch-Warner (PW) gravitational renormalization group flow [1] passes a number of important consistency checks to be identified as a holographic dual to a large- N $SU(N)$ $\mathcal{N} = 2^*$ supersymmetric gauge theory, it fails to reproduce the free energy of the theory on S^4 , computed with the localization techniques. This disagreement points to the existence of a larger dual gravitational consistent truncation, which in the gauge theory flat-space limit reduces to a PW flow. Such truncation was recently identified by Bobev-Elvang-Freedman-Pufu (BEFP) [2]. Additional bulk scalars of the BEFP gravitation truncation might lead to destabilization of the finite-temperature deformed PW flows, and thus modify the low-temperature thermodynamics and hydrodynamics of $\mathcal{N} = 2^*$ plasma. We compute the quasinormal spectrum of these bulk scalar fields in the thermal PW flows and demonstrate that these modes do not condense, as long as the masses of the $\mathcal{N} = 2^*$ hypermultiplet components are real.

KEYWORDS: Gauge-gravity correspondence, Black Holes in String Theory, Holography and quark-gluon plasmas

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1 Introduction and summary

In [1] Pilch and Warner (PW) proposed a holographic renormalization group flow dual to $\mathcal{N} = 2$ supersymmetric $SU(N)$ gauge theory, obtained by turning on a mass term for the $\mathcal{N} = 2$ hypermultiplet of the parental $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory. This massive deformation of $\mathcal{N} = 4$ SYM is commonly referred to as $\mathcal{N} = 2^*$ theory. As usual in most examples of gauge/gravity correspondence, the PW gravitational description is valid when the gauge theory is in the planar limit, and has large (strictly speaking infinitely large) 't Hooft coupling in the ultraviolet fixed point. While $\mathcal{N} = 2$ supersymmetric gauge theories have moduli space of vacua, the PW flow describes $\mathcal{N} = 2^*$ gauge theory at a particular point on a Coulomb branch [3]. Specifically, if Φ is an adjoint chiral multiplet (part of the $\mathcal{N} = 2$ vector multiplet), its Cartan subalgebra expectation values parameterize a generic Coulomb branch vacuum

$$\Phi = \text{diag}(a_1, a_2, \dots, a_N), \quad \sum_i a_i = 0, \quad a_i \in \mathbb{C}. \quad (1.1)$$

In the planar limit, $N \rightarrow \infty$ and $g_{YM}^2 \rightarrow 0$ with $g_{YM}^2 N$ kept fixed, it is natural to characterize the eigenvalue set (1.1) with a continuous density distribution, $\rho(a)$,

$$\int \int_{\mathbb{C}} d^2 a \, \rho(a) = N. \quad (1.2)$$

In [3] the PW vacuum was shown to be uniquely identified with the following eigenvalue distribution:

$$\begin{aligned} \text{Im}(a_i) &= 0, & a_i &\in [-a_0, a_0], & a_0^2 &= \frac{m^2 g_{YM}^2 N}{4\pi^2}, \\ \rho(a) &= \frac{8\pi}{m^2 g_{YM}^2} \sqrt{a_0^2 - a^2}, & \int_{-a_0}^{a_0} da \, \rho(a) &= N, \end{aligned} \quad (1.3)$$

where m is the hypermultiplet mass, and $\lambda \equiv g_{YM}^2 N \gg 1$ is a 't Hooft coupling of the parental $\mathcal{N} = 4$ SYM.

The question “*What makes the PW vacuum (1.3) special?*” was answered in [4]: turns out, supersymmetric formulation of $\mathcal{N} = 2^*$ gauge theory on S^4 , in the planar limit and at large ’t Hooft coupling, lifts all the Coulomb branch moduli, except for a single point; this single point, in the S^4 decompactification limit is precisely the vacuum (1.3). Furthermore, it was shown in [4] that the expectation value of the supersymmetric Wilson loop in PW geometry correctly reproduces the decompactification limit of the corresponding large- N matrix model [5] computation.

However, the agreement between the matrix model and the holographic computations does not extend to the free energy of the $\mathcal{N} = 2^*$ theory on S^4 [6]. Although a holographic renormalization produces a scheme-dependent result for the free energy,¹ it is possible to completely parameterize these free energy ambiguities [7, 8]. One can prove then that there does not exist a choice of a scheme in which the free energy of the S^4 -compactified PW flow would agree with the free energy of the $\mathcal{N} = 2^*$ theory computed in the large- N , and large λ , saddle point of the corresponding matrix integral [6]. It was suggested in [6] (also in [9–11]) that this free energy puzzle points to the existence of an “enlarged” holographic truncation of the $\mathcal{N} = 2^*$ gauge theory, which in addition to PW bulk scalars α and χ contains additional scalars representing the coupling of the gauge theory to S^4 background metric, necessary to insure the curved-space supersymmetry [5]. This alternative gravitational truncation was found in [2] (BEFP). At this stage the full ten-dimensional uplift of the BEFP effective action is unknown; nonetheless, BEFP five-dimensional action is enough to demonstrate that holographic free energy (in a certain renormalization scheme) agrees precisely with the matrix model localization result.

PW effective action, as a holographic dual to $\mathcal{N} = 2^*$ supersymmetric gauge theory, has been extensively used as a benchmark for gravitational computations of the thermodynamics [12–14], the hydrodynamics [15–18], and the entanglement entropy [19] of strongly coupled nonconformal gauge theory plasmas. For example, in [20] a critical phenomena in $\mathcal{N} = 2^*$ plasma was identified, which appears to be outside the dynamical universality classes established by Hohenberg and Halperin [21]. Existence of the alternative gravitational dual identified by BEFP raises the question as to what properties of the strongly coupled $\mathcal{N} = 2^*$ plasma can be reliably computed with PW effective action. From the gravitation perspective, the question is whether black hole solutions found in the framework of PW effective action are stable with respect to (normalizable) fluctuations of the additional BEFP bulk scalars.

In this paper we compute the spectrum of quasinormal modes of BEFP bulk scalars in PW black brane geometries. We show that as long as the masses of the hypermultiplet components are real, i.e., both the bosonic component m_b^2 , and the fermionic components m_f^2 , of the hypermultiplet mass-squared are positive, all quasinormal modes attenuate — the PW horizon bulk geometries are stable. Thus, the low-energy properties of $\mathcal{N} = 2^*$ plasma can be reliably computed from the holographic PW action. On the other hand, we find that there exist a regime, with tachyonic hypermultiplet component masses, where $SU(2)_V \subset SU(4)_R$ symmetry (see [2]) is spontaneously broken at sufficiently low temperatures.

¹The scheme dependence arises via the finite counterterms in the holographic renormalization.

The rest of the paper is organized as follows. In section 2 we compare BEFP and PW effective gravitational actions representing holographic dual to $\mathcal{N} = 2^*$ gauge theory at strong coupling. In section 3 we derive equations of motion and specify appropriate boundary conditions for BEFP normalizable fluctuations about thermal PW backgrounds. We compute the corresponding quasinormal spectra in two regimes:

- “bosonic” $\mathcal{N} = 4$ deformations: $m_b^2 \neq 0$, $m_f^2 = 0$;
- “supersymmetric” $\mathcal{N} = 4$ deformations: $m_b^2 = m_f^2 \equiv m^2$.

We find that while the thermal PW backgrounds are stable with respect to BEFP fluctuations for $m_b^2 > 0$ and $m^2 > 0$, the $SU(2)_V$ symmetry breaking fluctuations destabilize finite temperature PW flows with negative m_b^2 and m^2 at low temperatures. We conclude in section 4.

2 PW versus BEFP effective actions

We begin with description of the PW effective action [1], and its gravitational RG flows dual to $\mathcal{N} = 2^*$ plasma at strong coupling [12, 13]. The action of the effective five-dimensional gauged supergravity including the scalars α and χ (dual to mass terms for the bosonic and fermionic components of the hypermultiplet respectively) is given by

$$\begin{aligned} S &= \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \mathcal{L}_{\text{PW}} \\ &= \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left[\frac{1}{4} R - 3(\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P} \right], \end{aligned} \quad (2.1)$$

where the potential²

$$\mathcal{P} = \frac{1}{16} \left[\frac{1}{3} \left(\frac{\partial W}{\partial \alpha} \right)^2 + \left(\frac{\partial W}{\partial \chi} \right)^2 \right] - \frac{1}{3} W^2, \quad (2.2)$$

is a function of α and χ , and is determined by the superpotential

$$W = -e^{-2\alpha} - \frac{1}{2} e^{4\alpha} \cosh(2\chi). \quad (2.3)$$

In our conventions, the five-dimensional Newton’s constant is

$$G_5 \equiv \frac{G_{10}}{2^5 \text{vol}_{S^5}} = \frac{4\pi}{N^2}. \quad (2.4)$$

Regular horizon black brane solutions of (2.1) are found within the background ansatz:

$$\begin{aligned} ds_5^2 &= (2x - x^2)^{-1/2} e^{2a} \left(-(1 - x)^2 (dt)^2 + (d\vec{x})^2 \right) + g_{xx} (dx)^2, \\ \rho_6 &\equiv e^{6\alpha}, \quad c \equiv \cosh 2\chi, \end{aligned} \quad (2.5)$$

²We set the five-dimensional gauged supergravity coupling to one. This corresponds to setting the radius L of the five-dimensional sphere in the undeformed metric to 2.

where $\{a, \rho_6, c\}$ are functions of a radial coordinate

$$x \in [0, 1] . \quad (2.6)$$

The physical parameters of the plasma are encoded in the asymptotic coefficients of the metric and the bulk scalar functions near the boundary ($x \rightarrow 0_+$) and near the horizon ($x \rightarrow 1_-$). Specifically, as $x \rightarrow 0_+$ we have

$$\begin{aligned} \rho_6 &= 1 + 6x^{1/2} (\rho_{1,0} + \rho_{1,1} \ln x) + \mathcal{O}(x \ln^2 x) , \\ c &= 1 + x^{1/2} c_{1,0} + x \left(c_{2,0} + \frac{1}{3} c_{1,0}^2 \ln x \right) + \mathcal{O}(x^{3/2} \ln^2 x) , \\ a &= -\frac{1}{18} x^{1/2} c_{1,0} + \mathcal{O}(x \ln^2 x) , \end{aligned} \quad (2.7)$$

and as $y \equiv (1 - x) \rightarrow 0_+$ we have

$$\rho_6 = \rho_0 + \mathcal{O}(y^2) , \quad c = c_0 + \mathcal{O}(y^2) , \quad a = a_0 + a_1 y^2 + \mathcal{O}(y^4) . \quad (2.8)$$

The temperature and the hypermultiplet component masses are given by [13]

$$\begin{aligned} (\pi T)^2 &= e^{2a_0} \frac{4 + \rho_0^2 + 8\rho_0 c_0 - \rho_0^2 c_0^2}{48(1 + 4a_1)\rho_0^{2/3}} , \\ \left(\frac{m_b}{\pi T}\right)^2 &= 12\sqrt{2} e^{6a_0} \rho_{1,1} , \quad \left(\frac{m_f}{\pi T}\right)^2 = \sqrt{2} e^{6a_0} c_{1,0} . \end{aligned} \quad (2.9)$$

Note that in (2.9) we set $2\pi T = 1$ in the conformal case, i.e., when $\rho_6(x) = c(x) \equiv 1, a(x) \equiv 0$. This does not affect final results as long as we express them in dimensionless ratios (the temperature is one of the three microscopic mass scales in $\mathcal{N} = 2^*$ plasma, and for dimensionless results we can set one of these scales to an arbitrary value). The full T dependence can be restored via an arbitrary shift of the metric factor a . Furthermore, from (2.9), the supersymmetry condition, i.e., $m_b^2 = m_f^2$, constraints

$$c_{1,0} = 12\rho_{1,1} . \quad (2.10)$$

It was argued in [6, 9–11] that PW effective action (2.1) can not be the consistent gravitational truncation³ of the holographic dual to $\mathcal{N} = 2^*$ plasma — it fails to reproduce the exact field-theoretic computations of the free energy of the theory on S^4 . The correct gravitational dual was found in [2]. The BEFP effective action is given by

$$\begin{aligned} S_{\text{BEFP}} &= \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \mathcal{L}_{\text{BEFP}} \\ &= \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left[R - 12 \frac{(\partial\eta)^2}{\eta^2} - 4 \frac{(\partial\vec{X})^2}{(1 - \vec{X}^2)^2} - \mathcal{V} \right] , \end{aligned} \quad (2.11)$$

³It is a consistent truncation of type IIB supergravity.

with the potential

$$\mathcal{V} = - \left[\frac{1}{\eta^4} + 2\eta^2 \frac{1 + \vec{X}^2}{1 - \vec{X}^2} - \eta^8 \frac{(X_1)^2 + (X_2)^2}{(1 - \vec{X}^2)^2} \right], \quad (2.12)$$

where $\vec{X} = (X_1, X_2, X_3, X_4, X_5)$ are five of the scalars and η is the sixth. The symmetry of the action reflects the symmetries of the dual gauge theory [2]: the two scalars (X_1, X_2) form a doublet under the $U(1)_R$ part of the gauge group, while (X_3, X_4, X_5) form a triplet under $SU(2)_V$ and η is neutral. The PW effective action is recovered as a consistent truncation of (2.11) with

$$X_2 = X_3 = X_4 = X_5 = 0, \quad (2.13)$$

provided we identify the remaining BEFP scalars (η, X_1) with the PW scalars (α, χ) as follows

$$e^\alpha \equiv \eta, \quad \cosh 2\chi = \frac{1 + (X_1)^2}{1 - (X_1)^2}. \quad (2.14)$$

Note that once $m_f \neq 0$ (correspondingly $X_1 \neq 0$), the $U(1)_R$ symmetry is explicitly broken; on the contrary, $SU(2)_V$ remains unbroken in truncation to PW.

The goal of this paper is to study stability of gravitational solutions obtained in PW truncation of BEFP holographic dual to $\mathcal{N} = 2^*$ gauge theory. The effective action describing fluctuations of PW backgrounds within BEFP is obtained linearizing (2.11) in X_2, X_3, X_4, X_5 scalar fields:

$$\begin{aligned} \delta\mathcal{L} &\equiv \mathcal{L}_{\text{BEFP}} - \mathcal{L}_{\text{PW}} + \mathcal{O}(X_i^4) \equiv \delta\mathcal{L}_2 + \delta\mathcal{L}_V, \\ \delta\mathcal{L}_2 &= -(1+c)^2 (\partial X_2)^2 - \frac{1+c}{4} \left((c^2+c) \rho_6^{4/3} - 4(1+c) \rho_6^{1/3} + \frac{4(\partial c)^2}{c^2-1} \right) (X_2)^2, \\ \delta\mathcal{L}_V &= -(1+c)^2 (\partial \vec{X}_V)^2 - \frac{1+c}{4} \left((c^2-1) \rho_6^{4/3} - 4(1+c) \rho_6^{1/3} + \frac{4(\partial c)^2}{c^2-1} \right) (\vec{X}_V)^2, \end{aligned} \quad (2.15)$$

where $\vec{X}_V = (X_3, X_4, X_5)$. Note that $\delta\mathcal{L}$ is $SU(2)_V$ invariant; as a result it is enough to consider a spectrum of only one of \vec{X}_V components. In what follows we choose the latter to be X_3 .

3 Spontaneous breaking of $SU(2)_V$ symmetry in thermal PW flows

In this section we study the spectrum of normalizable fluctuations of (2.15) in thermal PW backgrounds (2.5)–(2.9). The framework for such analysis is equivalent to the one of the chiral symmetry breaking quasinormal modes in cascading gauge theory plasma developed in [22]. As mentioned in the introduction, we focus on bosonic ($m_f^2 = 0$) and supersymmetric $m_b^2 = m_f^2$ thermal flows — extensions to more general flows are straightforward, and we do not expect physically different results compared to the ones reported here.

We omit subscript for X whenever the discussion is identical for X_2 and X_3 BEFP scalars. Without the loss of generality we assume

$$X(t, \vec{x}, x) = e^{-i\omega t + i k x_3} F(x). \quad (3.1)$$

The radial wave-function $F(x)$ satisfies a homogeneous second order differential equation.⁴ The physical fluctuations must satisfy an incoming wave boundary condition at the PW black brane horizon, and be normalizable at the asymptotic $x \rightarrow 0_+$ boundary. Introducing

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \quad \mathfrak{q} = \frac{k}{2\pi T}, \quad (3.2)$$

the former condition implies

$$F(x) = (1-x)^{-i\mathfrak{w}} f(x), \quad (3.3)$$

with $f(x)$ being regular at the horizon, i.e., as $x \rightarrow 1_-$. The equation of motion for $f(x)$ is complex — it becomes real once we introduce

$$\mathfrak{w} = -i\Omega, \quad \text{Im}(\Omega) = 0. \quad (3.4)$$

Using the background asymptotic expansion (2.7), the normalizability condition for f at the $x \rightarrow 0_+$ boundary translates into the following asymptotic solutions,

- for the bosonic thermal flows:

$$\begin{aligned} f_2 &= f_{2,1,0} x^{3/4} \left(1 + \frac{x^{1/2}}{\sqrt{2}} \left((2\pi T \mathfrak{q})^2 + (2\pi T \Omega)^2 \right) + \mathcal{O}(x \ln^2 x) \right), \\ f_3 &= f_{3,1,0} x^{1/2} \left(1 + x^{1/2} \left(\sqrt{2}(2\pi T \mathfrak{q})^2 + \sqrt{2}(2\pi T \Omega)^2 + 8\rho_{1,1} - 2\rho_{1,0} - 2\rho_{1,1} \ln x \right) \right. \\ &\quad \left. + \mathcal{O}(x \ln^2 x) \right); \end{aligned} \quad (3.5)$$

- for the supersymmetric thermal flows:

$$\begin{aligned} f_2 &= f_{2,1,0} x^{3/4} \left(1 + \frac{x^{1/2}}{\sqrt{2}} \left((2\pi T \mathfrak{q})^2 + (2\pi T \Omega)^2 - 4\sqrt{2}\rho_{1,1} \right) + \mathcal{O}(x \ln^2 x) \right), \\ f_3 &= f_{3,1,0} x^{1/2} \left(1 + x^{1/2} \left(\sqrt{2}(2\pi T \mathfrak{q})^2 + \sqrt{2}(2\pi T \Omega)^2 + \frac{22}{3}\rho_{1,1} - 2\rho_{1,0} - 2\rho_{1,1} \ln x \right) \right. \\ &\quad \left. + \mathcal{O}(x \ln^2 x) \right). \end{aligned} \quad (3.6)$$

Since the equation of motion for f is homogeneous, without the loss of generality we can set $f(1) = 1$. The IR, i.e., as $y \equiv (1-x) \rightarrow 0_+$, asymptotic expansion then takes form

$$f = 1 + \mathcal{O}(y^2). \quad (3.7)$$

Notice that for each mode (f_2 or f_3) we have a single adjustable parameter: $f_{2,1,0}$ or $f_{3,1,0}$, in order to solve a boundary value problem for a second-order differential equations for f . As a result, a solution produces a dispersion relation for the BEFP fluctuations:

$$\Omega = \Omega(\mathfrak{q}^2). \quad (3.8)$$

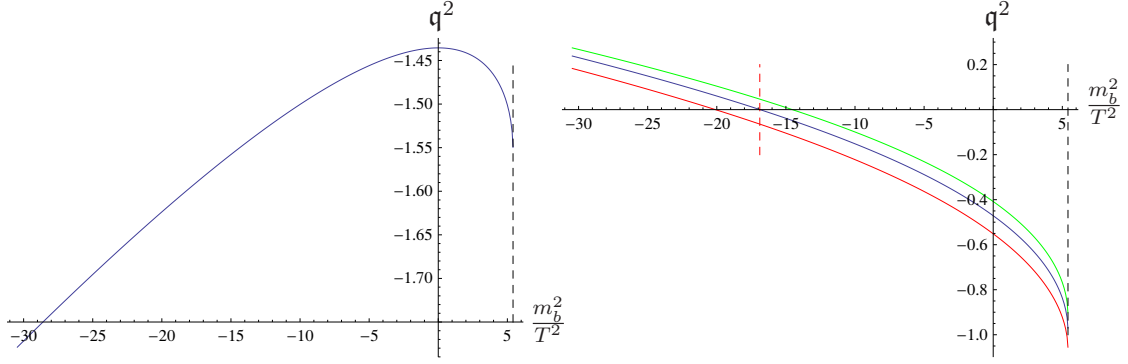


Figure 1. (Colour online) *Left panel:* dispersion relation of the BEFP fluctuations f_2 in the bosonic thermal PW flows as a function of $\frac{m_b^2}{T^2}$ at the threshold of instability: $(\mathfrak{w} = 0, q^2)$. *Right panel:* dispersion relation of the BEFP fluctuations f_3 in the bosonic thermal PW flows. The solid blue line represents the dispersion relation at the threshold of instability: $(\mathfrak{w} = 0, q^2)$. The solid green line indicates quasinormal modes with $(\Omega = 0.1, q^2)$ as a function of $\frac{m_b^2}{T^2}$; the solid red line indicates quasinormal modes with $(\Omega = -0.1, q^2)$ as a function of $\frac{m_b^2}{T^2}$. The red dashed vertical line indicates the onset of instability: for smaller values of $\frac{m_b^2}{T^2}$ the f_3 mode condenses with spontaneous breaking of $SU(2)_V$ symmetry of thermal bosonic PW flows. In *both panels*, the black dashed vertical lines represent the critical point in $\mathcal{N} = 2^*$ phase diagram, see [20].

The quasinormal modes signal an instability in plasma provided

$$\text{Im}(\mathfrak{w}) > 0 \Leftrightarrow \Omega < 0, \quad \text{provided} \quad \text{Im}(q) = 0. \quad (3.9)$$

The results of the analysis of the dispersion relation of BEFP fluctuations are presented in figures 1 (for bosonic thermal flows) and 2 (for supersymmetric thermal flows). In principle, we expect discrete branches of the quasinormal modes distinguished by the number of nodes in radial profiles f . In what follows we consider only the lowest quasinormal mode, which has monotonic radial profile.

The black dashed vertical lines in figure 1 represent the critical temperature T_c in $\mathcal{N} = 2^*$ plasma (see [20] for detailed discussion),

$$\frac{m_b}{T} \approx 2.32591. \quad (3.10)$$

The left panel presents the spectrum of f_2 fluctuations, and the right panel presents the spectrum of f_3 fluctuations. Solid blue lines indicate dispersion relations for BEFP fluctuations at the threshold of instability, i.e., with $\mathfrak{w} = 0$. Notice that on-shell fluctuations of f_2 mode have dispersion with $q^2 < 0$, implying that they are massive. Thus, BEFP mode f_2 never condenses in thermal bosonic PW flows. On the contrary, BEFP mode f_3 condenses with spontaneous breaking of $SU(2)_V$ symmetry of the PW truncation for

$$\frac{m_b^2}{T^2} < \frac{m_b^2}{T_{u,b}^2} \approx -16.8(9), \quad (3.11)$$

indicated by the red dashed vertical line in the right panel. At a given temperature, quasinormal modes with q^2 below the momenta of the modes at the threshold of instability

⁴These equations are too long to be presented here. They are available as supplement to the *arXiv.org* submission of this paper.

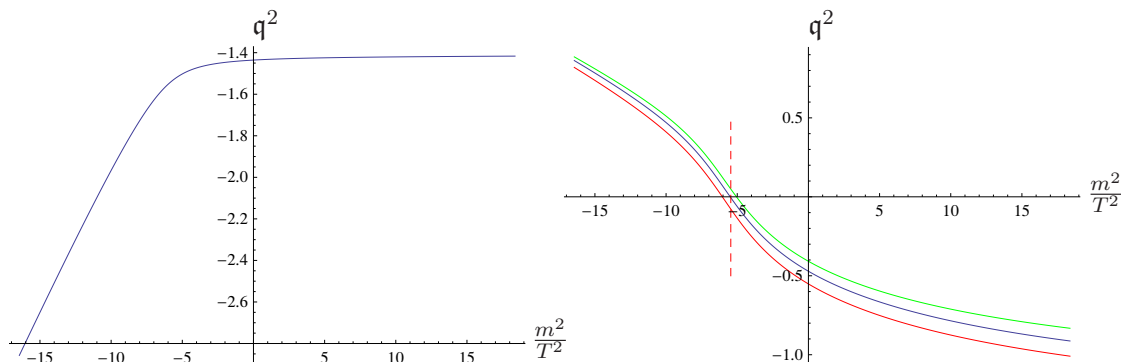


Figure 2. (Colour online) *Left panel:* dispersion relation of the BEFP fluctuations f_2 in the supersymmetric thermal PW flows as a function of $\frac{m^2}{T^2}$ at the threshold of instability: $(\mathfrak{w} = 0, q^2)$. *Right panel:* dispersion relation of the BEFP fluctuations f_3 in the supersymmetric thermal PW flows. The solid blue line represents the dispersion relation at the threshold of instability: $(\mathfrak{w} = 0, q^2)$. The solid green line indicates quasinormal modes with $(\Omega = 0.1, q^2)$ as a function of $\frac{m^2}{T^2}$; the solid red line indicates quasinormal modes with $(\Omega = -0.1, q^2)$ as a function of $\frac{m^2}{T^2}$. The red dashed vertical line indicates the onset of instability: for smaller values of $\frac{m^2}{T^2}$ the f_3 mode condenses with spontaneous breaking of $SU(2)_V$ symmetry of thermal supersymmetric PW flows.

(blue line) are expected to have $\Omega < 0$ (indicating a genuine tachyonic instability), while modes with q^2 above the momenta of the modes at the threshold of instability are expected to have $\Omega > 0$ (indicating stable excitations). This is precisely what we find: the red line in the right panel have $\Omega = -0.1$ and the green line indicate quasinormal modes with $\Omega = 0.1$.

The supersymmetric thermal PW flows exhibit identical pattern, see figure 2: BEFP mode f_2 is always stable, while BEFP mode f_3 condenses with spontaneous breaking of $SU(2)_V$ symmetry of the PW truncation for

$$\frac{m^2}{T^2} < \frac{m^2}{T_{u,s}^2} \approx -5.4(5). \quad (3.12)$$

4 Conclusion

The BEFP construction [2] resolves puzzling feature of the $\mathcal{N} = 2^*$ holography: the disagreement between the holographic free energy computation of the S^4 -compactified supersymmetric PW flows and the exact field theoretic computations of the free energy via localization. This is achieved by embedding the PW effective action as $SU(2)_V$ -invariant sector of “enlarged” five-dimensional gauged supergravity. Compare to PW effective action, BEFP action contains a triplet of $SU(2)_V$ bulk scalar fields, and one additional neutral scalar. These additional bulk scalars are necessary to model the coupling of $\mathcal{N} = 2^*$ gauge theory to background S^4 metric, as required by the supersymmetry [5].

In this paper we studied the stability of PW embedding within BEFP for thermal flows, previously used to study various thermodynamic and hydrodynamic properties of $\mathcal{N} = 2^*$ plasma. We demonstrated that the embedding is stable for all physical masses of the gauge theory hypermultiplet components. Interestingly, the $SU(2)_V$ symmetry can be spontaneously broken, but this occurs for the tachyonic masses of the $\mathcal{N} = 2$ hypermultiplet components, and at sufficiently low temperatures.

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